

# First-forbidden $\beta$ decay of $^{17}\text{N}$ and $^{17}\text{Ne}$

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(February 9, 2008)

## Abstract

It is shown that differences, due to charge-dependent effects, in the  $^{17}\text{N}$  and  $^{17}\text{Ne}$  ground-state wave functions account for the fact that the experimentally measured branch for the  $\beta^+$  decay of  $^{17}\text{Ne}$  to the first excited state of  $^{17}\text{F}$  is roughly a factor of two larger than expected on the basis of nuclear matrix elements which reproduce the corresponding  $\beta^-$  branch in the decay of  $^{17}\text{N}$ .

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By measuring positrons in coincidence with 495-keV  $\gamma$  rays de-exciting the  $1/2^+$  first-excited state of  $^{17}\text{F}$ , Borge *et al.* [1] have obtained a branch of 1.65(16)% for the first-forbidden  $\beta^+$  decay of  $^{17}\text{Ne}$  to the  $1/2^+$  state. This is a very interesting result because the measured branch is roughly a factor of two larger than expected on the basis of nuclear matrix elements which reproduce the corresponding  $\beta^-$  branch of 3.0(5)% [2,3] in the decay of  $^{17}\text{N}$ . Recently, Ozawa *et al.* [4] have confirmed the magnitude of the  $\beta$  branch in  $^{17}\text{Ne}$  decay, obtaining a value of 1.44(16)% by a method which utilizes a 32 MeV/A radioactive beam of  $^{17}\text{Ne}$ .

The  $\beta$ -decay rate is given by  $ft = 6170$  sec. For the  $1/2^- \rightarrow 1/2^+$  transitions of interest,  $f = f^{(0)} + f^{(1)}$  where the superscript refers to the spherical tensor rank of the  $\beta$ -decay operator. In general,  $f^{(0)}$  is much larger than  $f^{(1)}$  and, to a very good approximation,

$$f^{(0)} = I_0(\xi'v + \frac{1}{3}W_0w + \xi w')^2 \quad (1)$$

where  $\xi = \pm\alpha Z/2R$  for  $\beta^\mp$  decay, with  $Z$  the charge of the daughter nucleus and  $R = 3.499$  fm for  $A = 17$ , and

$$w = \lambda\sqrt{3}\hat{J}_f/\hat{J}_i\langle J_f T_f || ir[C_1, \sigma] \frac{1}{\sqrt{2}}\tau || J_i T_i \rangle C \quad (2)$$

$$\xi'v = -\lambda\sqrt{3}\hat{J}_f/\hat{J}_i\langle J_f T_f || \frac{i}{M}[\sigma, \nabla] \frac{1}{\sqrt{2}}\tau || J_i T_i \rangle C \lambda_{Ce}^2 \quad (3)$$

with  $C$  being the isospin Clebsch-Gordan coefficient and  $\lambda = 1.26$ . Energies are expressed in units of the electron rest mass and, with  $I_0$  the integrated phase-space factor for allowed decays divided by the square of the Compton wave length for the electron, the nuclear matrix elements are in fm. The matrix element  $w'$  is closely related to  $w$  and takes a value  $\sim 0.7w$  [5]. These expressions are based on a systematic expansion of the electron radial wave functions developed by Behrens and Bühring [6], the arcane notation for the nuclear matrix elements in first-forbidden decays being historical (see [5] for details and definitions of the rank-1 matrix elements).

Aside from the use of first-forbidden  $\beta$  decay as a spectroscopic tool, there has been great interest in rank-0 decays for two reasons. The first dates back to the suggestion [7]

that the matrix element  $\xi'v$  of the time-like piece of the axial current  $\gamma_5$  should be strongly enhanced by meson-exchange currents, largely one-pion-exchange. This enhancement is now well established at  $\sim 60\%$  for light nuclei [8] and even larger for heavy nuclei [9]. It is often taken into account, as is done below, by multiplying  $\xi'v$  by a factor  $\varepsilon_{mec}$ . The second reason relates to the similarity of the operators for parity-mixing and rank-0 first-forbidden  $\beta$ -decay [10]. As a result of these fundamental interests, a large literature exists on many aspects of first-forbidden  $\beta$  decay and parity-mixing in light nuclei. The present treatment of the  $^{17}\text{N}$  and  $^{17}\text{Ne}$  decays, first studied theoretically by Towner and Hardy [11], is based on a systematic study [12] of  $J^\pm \rightarrow J^\mp$  decays of  $^{11}\text{Be}$ ,  $^{15}\text{C}$ ,  $^{16}\text{C}$ ,  $^{16}\text{N}$ ,  $^{17}\text{N}$ ,  $^{18}\text{Ne}$ ,  $^{19}\text{Ne}$  and  $^{20}\text{F}$ .

For the  $1\hbar\omega$  basis used in [12], the  $1/2^-$  initial-state wave functions have a particularly simple form in a weak-coupling representation, namely that of a  $0p$ -shell hole coupled to  $(1s0d)^2$  eigenstates (notation  $J_n^\pi; T$ ).

$$\begin{aligned} |1/2^-; 3/2\rangle &= 0.967|1/2^- \otimes 0_1^+; 1\rangle - 0.224|3/2^- \otimes 2_1^+; 1\rangle \\ &+ 0.109|1/2^- \otimes 0_2^+; 1\rangle + \dots \end{aligned} \quad (4)$$

In fact, the three components listed account for 99.7% of the wave function. For the dominant component, only the  $1s_{1/2}^2$  component contributes to the matrix element  $\sigma \cdot \mathbf{r}$  and  $\sigma \cdot \mathbf{p}$ , one  $s_{1/2}$  nucleon making a transition to fill the  $p_{1/2}$  hole with the other forming the single-particle final state. The same is true for the third component, which augments the first (the  $0_2^+; 1$  state has a dominant  $1s_{1/2}^2$  component). A small  $d_{3/2} \rightarrow p_{3/2}$  amplitude, arising from the second component of the  $1/2^-$  wave function, is important because the single-particle matrix element is large (larger than  $s \rightarrow p$  by a factor of  $\sqrt{5}$  for harmonic oscillator wave functions) and interferes destructively with the dominant  $1s_{1/2} \rightarrow 0p_{1/2}$  amplitude. This is a common feature of all the transitions studied in [12]. The radial single-particle matrix elements are computed with Woods-Saxon wave functions obtained by adjusting the well depth to match the separation energy from the initial or final state to the appropriate physical core states of the A-1 system [12]. For the  $1s_{1/2} \rightarrow 0p_{1/2}$  contribution, the only important parent states are the lowest  $0^-$  and  $1^-$  states of  $^{16}\text{N}$  or  $^{16}\text{F}$ . The separation energies are given in Table I

along with the decay energies and the phase-space integrals  $I_0$ . Since the separation energies are close to the Hartree-Fock energies, the Woods-Saxon wave functions should be a good approximation to one-nucleon overlap functions [13].

For the rank-0 contribution to the  $\beta$ -decay rates, the calculation gives

$$f^{(0)}(\text{N}) = 0.3051(10.971\varepsilon_{mec} - 4.216)^2 \quad (5)$$

$$f^{(0)}(\text{Ne}) = 2.380(11.585\varepsilon_{mec} - 3.009)^2 \quad (6)$$

$$f^{(0)}(\text{Ne}') = 2.380(15.278\varepsilon_{mec} - 3.969)^2 \quad (7)$$

where the first two lines correspond to using identical nuclear structure, the small differences in matrix elements being due to the use of Woods-Saxon wave functions bound at the physical separation energies (note the energy-dependent factors in Eq. (1) for the second term). The resulting  $f$  values are compared with experiment in Table II for two values of the enhancement due to meson-exchange currents (see Table IV of [8] for theoretical estimates of  $\varepsilon_{mec}$ ). Including the calculated  $f^{(1)}$  values, it can be seen that the predicted value for the  $\beta$  branch in  $^{17}\text{Ne}$  is less than  $\sim 0.9\%$  for values of  $\varepsilon_{mec}$  which produce agreement with the  $^{17}\text{N}$  data (0.77% to reproduce the central value).

For the case denoted by  $\text{Ne}'$  in Eq. (7) and the last line of Table II, the  $^{17}\text{Ne}$  ground-state wave function has been modified to take into account charge-dependent effects which differ for  $1s$  and  $0d$  orbits. Now, with a 45 – 50% enhancement from meson-exchange currents, the calculated beta-decay rates are in agreement, within the error bars, for both nuclei.

That there should be substantial  $T_z$ -dependent effects is evident from the 376 keV difference in Coulomb energies for the  $0d_{5/2}$  and  $1s_{1/2}$  orbits at  $A = 17$ . For  $A = 18$ , the large shift in the excitation energy of the the third  $0^+$  state in  $^{18}\text{Ne}$  (Table III) led to its identification as a largely  $1s_{1/2}^2$  configuration [14]. The shift in the  $s_{1/2}^2$  diagonal matrix element relative to  $d_{5/2}^2$  in going from  $^{18}\text{O}$  to  $^{18}\text{Ne}$  will also lead to more  $s_{1/2}^2$  in the  $^{18}\text{Ne}$  ground-state wave function and hence, when coupled to a  $p_{1/2}$  hole, to an enhancement of the rank-0 matrix element for the  $\beta^+$  decay of  $^{17}\text{Ne}$ . This effect is amplified by the cancellation between the  $s_{1/2} \rightarrow p_{1/2}$  and  $d_{3/2} \rightarrow p_{3/2}$  contributions.

To make a rough estimate of this effect, the Wildenthal USD interaction [15] is used to obtain  $(sd)^2$  wave functions for  $^{18}\text{O}$  ( $\epsilon_{5/2} = -3.9478$ ,  $\epsilon_{1/2} = -3.1635$ ,  $\epsilon_{3/2} = 1.6466$ , upper half diagonal of two-body matrix elements -2.8197, -1.3247, -3.1856, -2.1246, -1.0835, -2.1845). Then, the  $s_{1/2}^2$  diagonal matrix element is shifted by twice the shift of the  $s_{1/2}$  single-particle energies between  $^{17}\text{O}$  and  $^{17}\text{F}$  (752 keV) plus 147 keV for the difference between the two-body matrix elements of  $e^2/r$  for  $d^2$  and  $s^2$  configurations [16], and the new matrix is diagonalized to get  $(sd)^2$  wave functions for  $^{18}\text{Ne}$ . The resulting energies, wave functions, and intensities of  $1s_{1/2}^2$  are given in Table IV. The  $s_{1/2}^2$  intensity rises from 15% to 21.7%, an increase of 44% (the squared overlap of the ground-state wave functions is still 0.9925). The increase in  $\xi'v$  in Eq. (7) by a factor 1.32 rather than 1.20 for the  $s_{1/2} \rightarrow p_{1/2}$  matrix element alone is due to the cancellation effects involving the  $d_{3/2} \rightarrow p_{3/2}$  matrix element.

The above calculation, which does succeed in providing an explanation for the measured  $\beta$ -decay rates, is not a consistent one but clearly indicates the direction in which charge-dependent effects will affect the  $\beta$ -decay branch in  $^{17}\text{Ne}$  decay. An explanation of the energy shifts and wave function changes for the  $0^+ T = 1$  states of  $A = 18$  requires that the  $4p2h$  configurations be included. A calculation of the energy shifts without wave function changes [14] does rather well but the  $^{18}\text{Ne}$  ground state could do with a “push” of the magnitude (163 keV) shown in Table IV. The  $(sd)^2$  calculation is actually more applicable to the  $2p1h$  states of  $^{17}\text{N}$  and  $^{17}\text{Ne}$  because the  $4p3h$  states are expected [17] to lie above both states obtained by coupling a  $p_{1/2}$  hole to the two lowest  $(sd)^2 0^+$  states. The second of these states is known at 3.663 MeV in  $^{17}\text{Ne}$  and is lowered from its position in  $^{18}\text{O}$  in large part because the spin-average  $p_{1/2}^{-1}s_{1/2} T = 1$  particle-hole interaction is less repulsive by  $\sim 700$  keV than the corresponding  $p_{1/2}^{-1}d_{5/2}$  interaction [18,17] and to a lesser extent because of the removal of the influence of the  $4p2h$  configuration.

To put the structure of  $^{17}\text{N}$  and  $^{17}\text{Ne}$  in a broader context, it should be noted that the four particle-hole matrix elements mentioned above can be deduced directly from the binding energies of the lowest four states of  $^{16}\text{N}$  (the charge-dependent shifts of the  $0d_{5/2}$  and  $1s_{1/2}$  orbits, including a dependence on separation energy, can be seen across these  $T = 1$

multiplets). Within the framework of the same weak-coupling assumption used to deduce the particle-hole matrix elements, the total binding energies and multiplet spacings of the low-lying states of the heavy carbon and nitrogen isotopes which contain one or more  $sd$ -shell neutrons can be rather nicely accounted for (of course, small components in the wave functions are important for detailed spectroscopic applications such as first-forbidden  $\beta$  decay). In consistent shell-model calculations which include charge-dependent interactions, the response to changes in  $T_z$  on the one hand and to changes in the number of particles or holes on the other strongly restricts the  $d_{5/2}/s_{1/2}$  content of the low-lying states. An interesting case in the context of the present study is  $^{16}\text{C}$  which has a rank-0  $\beta$  decay branch of 0.68% [19] to the lowest  $0^-$  state of  $^{16}\text{N}$ . With an extra  $p_{1/2}$  proton hole, the energy of the excited  $0^+$  state has been lowered to 3.02 MeV, implying slightly more  $1s_{1/2}^2$  in the ground state than for  $^{17}\text{N}$ . The first-forbidden  $\beta$ -decay rate is well accounted for using the same type of shell-model calculation and meson-exchange enhancement as for  $^{17}\text{N}$  [12].

A unique first-forbidden  $\beta$  branch of 1.6(5)% [20] to the ground state of  $^{17}\text{O}$  is known for the decay of  $^{17}\text{N}$ . This branch corresponds to  $f^{(2)} = 24(8)$ . With no change in the single nuclear matrix element involved, the expected branch in  $^{17}\text{Ne}$  decay is 0.55(18)%. Charge-dependent effects should lower this value slightly because of a decrease in the  $d_{5/2}^2$  component of the  $^{17}\text{Ne}$  ground state (Table IV), amplified somewhat by cancellation between  $d_{5/2} \rightarrow p_{1/2}$  and  $d_{5/2} \rightarrow p_{3/2}$  contributions. Shell-model calculations with the basis of Ref. [12] overpredict  $f^{(2)}$  by a little more than a factor of two for either harmonic oscillator or Woods-Saxon wave functions. This is quite consistent with a similar overestimate for the unique first-forbidden decay of  $^{16}\text{N}$  for a correspondingly small shell-model basis. This problem is resolved in calculations using a very large shell-model basis with all configurations up to  $4\hbar\omega$  [8,21]. The rank-0 matrix elements are also reduced in such calculations [8] but by a lesser amount due to a cancellation between contributions from  $2p2h$  admixtures induced by central and tensor forces. The experimental  $\beta$ -decay rates can then be reproduced using values for  $\varepsilon_{mec}$  close to the theoretical value of about 1.6 [8].

In conclusion, the use of realistic (e.g., Woods-Saxon) radial wave functions is essential

for evaluating first-forbidden  $\beta$ -decay matrix elements [8,12], particularly for  $1s_{1/2} \leftrightarrow 0p_{1/2}$  transitions for which the  $1s_{1/2}$  nucleon is loosely bound, as is the case for the decay of  $^{17}\text{N}$  and  $^{17}\text{Ne}$  to the first-excited  $1/2^+$  states of  $^{17}\text{O}$  and  $^{17}\text{F}$ . However, radial wave function differences do not account for the strong asymmetry observed for these decays. Rather, plausible  $T_z$ -dependent differences in the  $1s_{1/2}$  occupancy for the initial states can account for the asymmetry. Furthermore, the very small separation energy for the  $1s_{1/2}$  proton in  $^{17}\text{F}$  is not germane to the problem since this proton is a spectator in the  $\beta$ -decay process. In fact, from the way in which the parentage expansion is made and separation energies determined, the spectator  $1s_{1/2}$  proton forms part of a  $^{16}\text{F}$  core where it is unbound for the physical core states (by 535 keV for the  $0^-$  state). Substantial asymmetries have also been observed for the allowed decays of  $^{17}\text{N}$  and  $^{17}\text{Ne}$  [22]. While overlap factors for radial wave functions bound at different energies now play a role because the Gamow-Teller operator has no spatial structure, it again seems likely that the observed asymmetries are largely due to  $T_z$ -dependent mixing of various shell-model configurations. For the  $2p1h$  configurations with  $T = 1/2$ , the mixing of configurations with  $T = 0$  and  $T = 1$  for the  $(sd)^2$  configurations determines both the overall spatial symmetry and the relative contributions to the Coulomb energy from  $p$  and  $sd$  orbits. There are also low-lying  $4p3h$  configurations (one  $1/2^-$  and two  $3/2^-$ ) which have their own Coulomb energy shifts and mix strongly with the  $2p1h$  configurations. Thus, there should be significant  $T_z$ -dependent mixing in both the initial and final states for the Gamow-Teller decays. A beautiful demonstration of this type of  $T_z$ -dependent mixing is seen in changes of the ratio of Gamow-Teller strengths for the lowest two  $2^+; T = 1$  states reached via  $(n, p)$ ,  $(p, p')$  and  $(p, n)$  reactions on  $^{14}\text{N}$  [23]. Here, the near degeneracy of  $2h$  and  $2p4h$  configurations [24], with Coulomb energies that differ by  $\sim 700$  keV across the multiplet, leads to very different wave functions for each nucleus.

This research was supported by the U. S. Department of Energy under Contract No. DE-AC02-76CH00016 with Brookhaven National Laboratory.

## REFERENCES

- [1] M. J. B. Borge, J. Deding, P. G. Hansen, B. Jonson, G. Martinez Pinedo, P. Møller, G. Nyman, A. Poves, A. Richter, K. Riisager, and O. Tengblad, Phys. Lett. B **317**, 25 (1993).
- [2] A. R. Poletti and J. G. Pronko, Phys. Rev. C **8**, 1285 (1973).
- [3] D. E. Alburger and D. H. Wilkinson, Phys. Rev. C **13**, 835 (1976).
- [4] A. Ozawa, M. Fujimaki, S. Fukuda, S. Ito, T. Kobayashi, S. Momota, T. Suzuki, I. Tanihata, K. Yoshida, H. Kitigawa, G. Kraus, and G. Münzenberg, preprint RIKEN-AF-NP-238.
- [5] D. J. Millener, D. E. Alburger, E. K. Warburton, and D. H. Wilkinson, Phys. Rev. C **26**, 1167 (1982).
- [6] H. Behrens and W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon, Oxford, England, 1982).
- [7] K. Kubodera, J. Delorme, and M. Rho, Phys. Rev. Lett. **40**, 755 (1978).
- [8] E. K. Warburton, I. S. Towner, and B. A. Brown, Phys. Rev. C **49**, 824 (1994).
- [9] E. K. Warburton and I. S. Towner, Phys. Rep. **243**, 103 (1994).
- [10] E. G. Adelberger and W. C. Haxton, Annu. Rev. Nucl. Part. Sci. **35**, 501 (1985).
- [11] I. S. Towner and J. C. Hardy, Nucl. Phys. **A179**, 489 (1972).
- [12] D. J. Millener and E. K. Warburton, *Nuclear Shell Models*, edited by M. Vallieres and B. H. Wildenthal (World Scientific, Singapore, 1985), p. 365.
- [13] G. R. Satchler, *Direct Nuclear Reactions* (Clarendon, Oxford, England, 1983), Sects. 16.4 and 17.4.
- [14] A. V. Nero, E. G. Adelberger, F. S. Dietrich, and G. E. Walker, Phys. Rev. Lett. **32**,



- 623 (1974); A. V. Nero, E. G. Adelberger, and F. S. Dietrich, Phys. Rev. C **24**, 1864 (1981).
- [15] B. H. Wildenthal, *Progress in Particle and Nuclear Physics*, edited by D. H. Wilkinson (Pergamon, Oxford, England, 1984), Vol. 11, p. 5.
- [16] S. H. Kahana, Phys. Rev. C **5**, 63 (1972).
- [17] E. K. Warburton and D. J. Millener, Phys. Rev. C **39**, 1120 (1989).
- [18] F. C. Barker, Aust. J. Phys. **37**, 17 (1984).
- [19] C. A. Gagliardi, G. T. Garvey, N. Jarmie, and R. G. H. Robertson, Phys. Rev. C **27**, 1353 (1983).
- [20] M. G. Silbert and J. C. Hopkins, Phys. Rev. **134**, B16 (1964).
- [21] E. K. Warburton, B. A. Brown, and D. J. Millener, Phys. Lett. B **293**, 7 (1992).
- [22] D. R. Tilley, H. R. Weller, and C. M. Cheves, Nucl. Phys. **A564**, 1 (1993).
- [23] K. P. Jackson for the CHARGEEX collaboration, private communication.
- [24] E. K. Warburton and W. T. Pinkston, Phys. Rev. **118**, 733 (1960).

# TABLES

TABLE I. Parameters governing the decays of  $^{17}\text{N}$  and  $^{17}\text{Ne}$  to the first-excited states of  $^{17}\text{O}$  and  $^{17}\text{F}$ . Separation energies are given for the  $0_1^-; 1$  core states in  $^{16}\text{N}$  and  $^{16}\text{F}$ ; the values for the  $1_1^-$  core state are 0.28 MeV and 0.19 MeV higher, respectively.

	$W_0$ (MeV)	$I_0$	$S_{n/p}(s_{1/2})$ (MeV)	$S_{p/n}(p_{1/2})$ (MeV)
$^{17}\text{N}$	8.32 MeV	0.3051	6.00 ( <i>n</i> )	13.03 ( <i>p</i> )
$^{17}\text{Ne}$	13.52 MeV	2.380	1.48 ( <i>p</i> )	16.80 ( <i>n</i> )

TABLE II. Comparison of theoretical and experimental  $\beta$ -decay rates via  $f$  values.  $f_{exp}$  for  $^{17}\text{Ne}$  decay is derived from the average 1.55(12)% of the two measurements [1,4] for the  $\beta$  branch.

	$f^{(0)}$	$f^{(1)}$	$f_{exp}$
$\varepsilon_{mec}$	1.4	1.5	
$^{17}\text{N}$	37.9	45.7	6.5
$^{17}\text{Ne}$	415	491	21
$^{17}\text{Ne}'$	722	854	21

TABLE III. Excitation energies (MeV) of  $0^+$   $T = 1$  states relative to the lowest such state. The  $0_2^+$  states are mainly  $4p2h$  in nature. In the case of  $^{18}\text{F}$ , it should be noted that the lowest  $0^+$  state obtains extra binding energy from the charge-independence breaking  $np$  interaction [16].

$J_n^\pi$	$^{18}\text{O}$	$^{18}\text{F}$	$^{18}\text{Ne}$
$0_3^+$	5.336	5.094	4.590
$0_2^+$	3.630	3.711	3.576

TABLE IV. Results of  $(sd)^2$  diagonalizations. Wave function amplitudes are given in columns 4 – 6. The binding energy of the  $0_1^+$  state of  $^{18}\text{O}$  is chosen as the zero of energy.

	$J_n^\pi$	$E_x$	$d_{5/2}^2$	$s_{1/2}^2$	$d_{3/2}^2$	$\%s_{1/2}^2$
$^{18}\text{O}$	$0_1^+$	0.000	0.8886	0.3878	0.2448	15.0
	$0_2^+$	4.320	0.3932	-0.9190	0.0287	84.5
$^{18}\text{Ne}$	$0_1^+$	-0.163	0.8521	0.4654	0.2394	21.7
	$0_2^+$	3.588	0.4667	-0.8827	0.0547	77.9